Key

Math 4 Name_______________

5-4 The Derivative Function

Date________

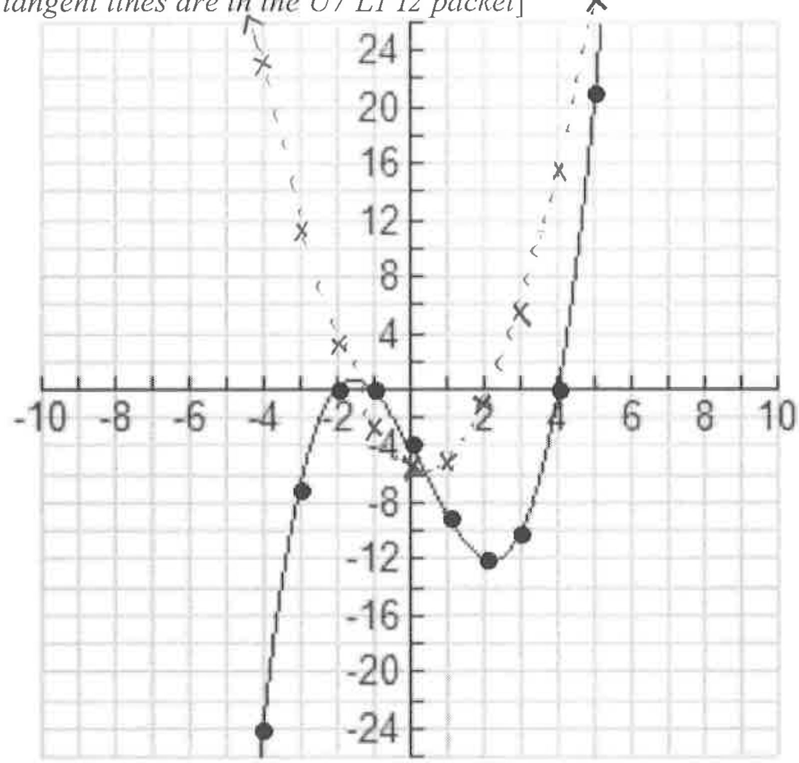
In this Activity, you will be working towards the following learning goals:
I can compute derivatives using both the definition and the power rule
I can use derivatives and their graphs to identify properties of functions

I. **Definition:** Suppose that f is a function that has a derivative $f'(x)$ at each point x in the domain of f. Then the function f': $x \to f'(x)$ for all x in the domain of f is called the **derivative function of f.**

$f'(x) = 1.5x^2 - x - 5$ (they won't know this yet)

Example #1: Consider the cubic function $f(x) = 0.5x^3 - 0.5x^2 - 5x - 4$. Use your calculator to find the equation of the tangent lines at the given points on the graph below (all points have integer coordinates), then record the value of $f'(x)$ in the table below.
[Directions for finding tangent lines are in the U7 L1 I2 packet]

x	$f'(x)$
−4	23
−3	11.5
−2	3
−1	−2.5
0	−5
1	−4.5
2	−1
3	5.5
4	15
5	27.5



Plot the 10 ordered pairs on the grid & connect them with a smooth curve. Answer the following questions:

1. What is the degree of $f(x)$? $\boxed{3}$
2. What type of function does $f'(x)$ appear to be? What is its degree?

   Quadratic. Degree of 2.

3. What type of function do you *think* $f''(x)$ would be? What is its degree?

   Linear. Degree of 1.

Based on questions 1 – 3, what happens to the degree each time you take the derivative of a function?

It decreases by one.

II. **Theorem: The Derivative of a Quadratic Function:**

If $f(x) = ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$, then $f'(x) = 2ax + b$ for all real numbers x. *(*** "x" is the variable.)*

Proof: $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x+\Delta x)^2 + b(x+\Delta x) + c - (ax^2 + bx + c)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a(x^2 + 2x\Delta x + \Delta x^2) + bx + b\Delta x + c - ax^2 - bx - c}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{ax^2} + 2ax\Delta x + a\Delta x^2 \cancel{+bx} + b\Delta x \cancel{+c} \cancel{-ax^2} \cancel{-bx} \cancel{-c}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2ax\cancel{\Delta x} + a\Delta x^{\cancel{2}} + b\cancel{\Delta x}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 2ax + a\Delta x + b$$

$$= \boxed{2ax + b}$$

Notes:

- Exponents decrease by one
- Mult. exponent by the coefficient in that term
- terms with no x disappear

Answer: $f'(x) = 2ax + b$

Example #2: Consider the quadratic function $f(x) = x^2 + 4$ used in Investigation 2.

a. Find $f'(x)$ algebraically, using the algebraic definition of derivative.

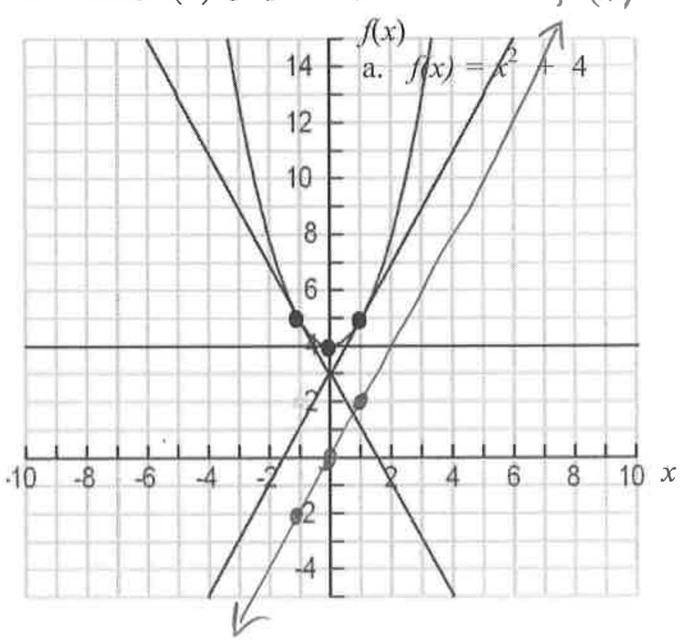
$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 4 - (x^2+4)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^2} + 2x\Delta x + \Delta x^2 + \cancel{4} - \cancel{x^2} - \cancel{4}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x \to 0} 2x + \Delta x$$

$$= \boxed{2x}$$

b. Find $f'(x)$ graphically.



x	$f'(x)$
-1	-2
0	0
1	2

c. Find $f'(x)$ using the new theorem.

$f'(x) = 2x$

****The theorem for the derivative of a quadratic function can be extended to linear and constant functions.*

If $f(x) = ax^2 + bx + c$, then $f'(x) =$ $2ax + b$.

Example #3: Find the derivative of $f(x) = 3x + 2$.

$a =$ 0 $b =$ 3 $c =$ 2

$f'(x) =$ 3

Example #4: Find the derivative of $f(x) = 5$.

$a =$ 0 $b =$ 0 $c =$ 5

$f'(x) =$ 0

Example #5: Verify that the circumference of a circle is the derivative of its area.

$C = 2\pi r$ $A = \pi r^2$

$A' = 2\pi r$ ✓

~~Notes:~~

Practice: Find the derivative of the following functions:

$f(x) = 4x^2 - 3x + 8$

$f'(x) = 8x - 3$

$g(x) = 5x^3 - 6x^2 - 2x + 6$

$g'(x) = 15x^2 - 12x - 2$

$h(x) = -x^4 + 2x^3 - x + 1$

$h'(x) = -4x^3 + 6x^2 - 1$

$j(x) = 4x^{-2} - 3$

$j'(x) = -8x^{-3} = \boxed{\dfrac{-8}{x^3}}$

$k(x) = \dfrac{1}{x^3} - \dfrac{3}{x^2} = x^{-3} - 3x^{-2}$

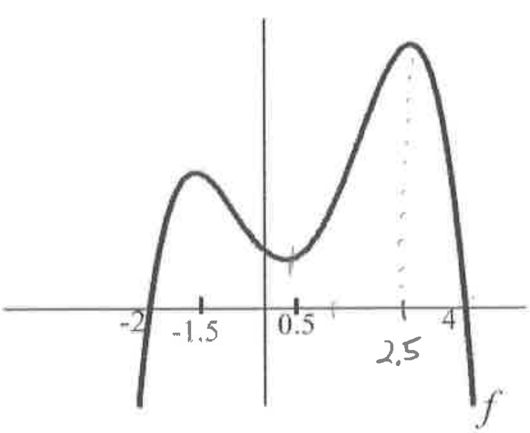
$k'(x) = -3x^{-4} + 6x^{-3}$

$= \boxed{\dfrac{-3}{x^4} + \dfrac{6}{x^3}}$

$m(x) = -4x^5 + \dfrac{1}{x^3} - 7x$

$m'(x) = -20x^4 - \dfrac{3}{x^4} - 7$

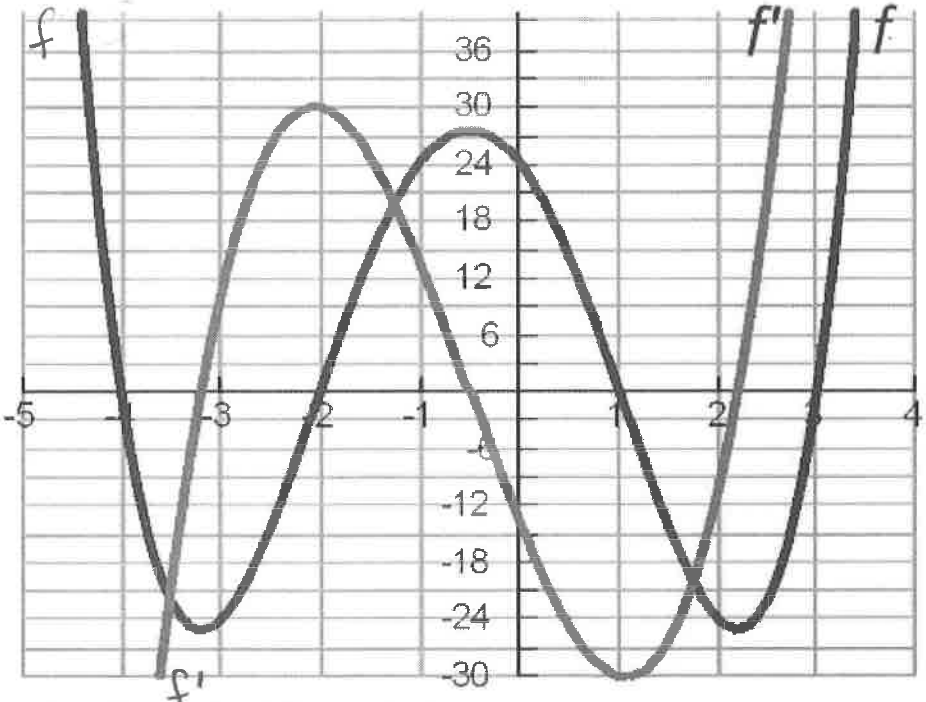
Below is the graph of $f(x)$. On what intervals is $f'(x) > 0$?



$$\boxed{\begin{aligned} &-2 \le x < -1.5 \\ &0.5 \le x < 2.5 \end{aligned}}$$

III. The Graph of the Derivative

Below is a graph of a function *f* and its derivative *f* '.



1a. On what intervals is the graph of *f* increasing?

$-3.2 < x < -0.5$, $x > 2.2$

1b. What is happening to the derivative when the graph of *f* is increasing?

$f'(x)$ is >0 (positive) at those x-values

2a. On what intervals is the graph of *f* decreasing?

$x < -3.2$ $-0.5 < x < 2.2$

2b. What is happening to the derivative when the graph of *f* is decreasing?

$f'(x)$ is <0 (negative) at the x-values where f is decreasing.

3a. Where are the local/global minimums and maximums or the graph of *f*?

Mins: $(-3.2, -25)$ + $(2.2, -25)$

Max: $(-0.5, 25)$

3b. What is happening to the derivative when the graph of *f* is at a local/global minimum or maximum?

The derivative has x-intercepts at those x-values.

4. Explain why your answers to 1b, 2b, and 3b make sense. When f(x) increases, slope is pos. f(x) decreases,

The derivative is the slope at each point of f(x).